First Ten Ångstroms

Dilational Stress Pump Waveforms

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Dilational stress analysis measures the "dose-response" or "stress-strain" between the surface area of the drop and the interfacial tension of the liquid. However, real pumps directly control drop volume, not drop surface area. The analysis assumes a sinusoidal variation of surface area, but a sinusoidal drop volume does *not* yield a sinusoidal drop area. As a simple example, assume for the moment that the drop was a sphere of radius r. Then its

Volume = $4/3 \pi r^3$

Surface Area = $4 \pi r^2$

so we can see that its surface area is *not* going to exactly track its volume, as the exponent of the radius term is only 2/3 of the volume term. More importantly, the precise volume change that would yield an exact sinusoidal surface area is unknowable *a priori* because of the distortion in shape from gravity (and, of course, that distortion is necessary to apply the Laplace-Young equation in the first place!).

The best test of the "quality" of the surface area waveform is Fourier analysis. The Fourier spectrum will contain a peak at the desired perturbation frequency, called the fundamental frequency, and two kinds of extraneous peaks:

- background noise, which will be spread more or less uniformly through the spectrum and is the result of vibration and electronic noise, and

- harmonic distortion which will be at an integer multiple of the fundamental in frequency and which represents the regular, repeatable, imperfections in the waveform.

Thus harmonic peaks in the surface area waveform provide a convenient "litmus test" for the waveform.

The triangle waveform is easy to generate using stepper motor controlled pumps as it consists of a linear ramp one way and then a linear ramp the other. The triangle can also run rapidly and it requires a minimum of changes in the stepper motor controller during each cycle.

Now the relevant question is: How good is it? It turns out the triangle in volume provides a good approximation to a sinusoid in surface area. This can be seen by inspection of surface area Fourier transforms for triangle volume waveforms, where little if any harmonic distortion can be seen. As an example, see *"Milk as an Example of Dynamic Interfacial Tension and Dilational Stress Measurements"* in the Papers section of the FTA website.

There is also an intuitive explanation for this good performance. The relationship between surface area and volume is of the order of volume raised to the 2/3 power. The exact exponent depends on the distortion from gravity, but the exponent is distinctly less than 1. An exponent less than one means the surface area waveform, compared to the volume waveform, will *have its peaks diminished and its valleys filled in a bit*. If this is hard to visualize, consider the opposite, an exponent greater than 1. In this case, the peaks would be accentuated, made higher and more pointy. So the exponent less than one is the other way: diminished peaks. This diminishing of the peaks rounds them off and makes the triangle look, as an area, more sinusoidal.

In summary, all we can hope for is a reasonable approximation to a surface area sinusoid, but the Fourier transform will tell use (after the fact) how well we did. Fourier analysis has the additional advantage of making it trivial to extract the fundamental sinusoidal component of the surface area and IFT waveforms, and furthermore these extracted peaks represent the best fit of a sinusoid to the actual waveforms.

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